4. Vzdálenost bodu od přímky a od roviny

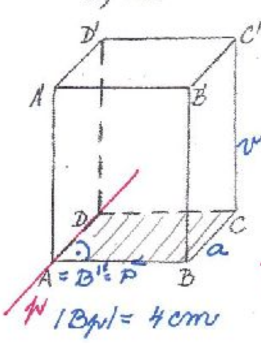
- VZDÁLENOST BODŮ A,B: délka úsečky AB, ozn. $|AB|$
- VZDÁLENOST BODU OD PŘÍMKY
  - určujeme jako vzdálenost bodu od přímky v rovině, protože bod a přímka v prostoru leží v jedné rovině
  - vzdálenost tohoto bodu od jeho pravoúhlého průmětu na tuto přímku
  - ozn. $|Ap| = |AP|$

- délka úsečky AP, kde P pata kolmice k vedené v rovině Ap bodem A k přímce p
  (nejmenší vzdál. všech bodů dané přímky od daného bodu)
- pro $A \in p \Rightarrow |Ap| = 0$

Příklady

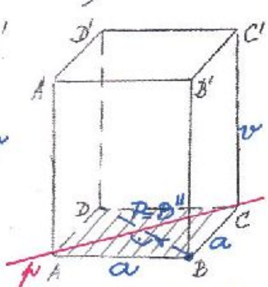
① Pravid. čtyřboký hranol $ABCDA'B'C'D'$, $|AB| = a = 4$ cm, $|AA'| = v = 5{,}5$ cm, M střed $A'D'$. Urči vzdálenost bodu B od přímky $B'' = P$... pata kolmice k p

a) AD



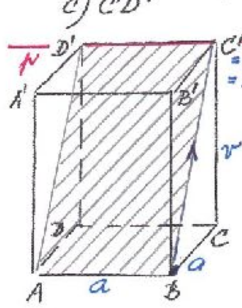
$|Bp| = 4$ cm

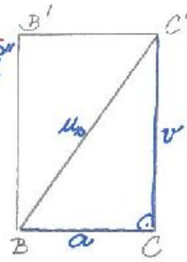
b) AC



$|Bp| = |BP| = \frac{u}{2} = \frac{a\sqrt{2}}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2} \doteq 2{,}8$ cm

c) C'D'





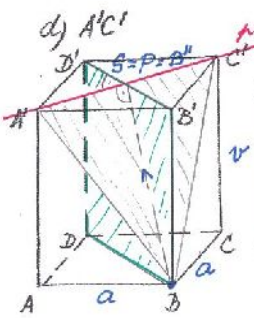
$|Bp| = |BC'| = u_s$

$= \sqrt{v^2 + a^2} = \sqrt{5{,}5^2 + 4^2}$

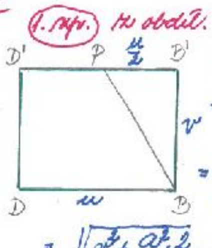
$|Bp| \doteq 6{,}8$ cm

$[u_s^2 = v^2 + a^2]$

d) A'C'



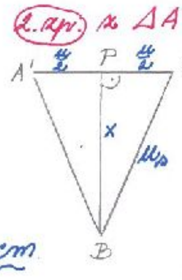
1.zp. z obdél. $BB'D'D$



$|Bp| = |BP|$

$= \sqrt{v^2 + \left(\frac{u}{2}\right)^2} = \sqrt{v^2 + \left(\frac{a\sqrt{2}}{2}\right)^2} = \sqrt{v^2 + \frac{a^2 \cdot 2}{2^2}} = \sqrt{v^2 + \frac{a^2}{2}} \doteq 6{,}2$ cm

2.zp. z $\Delta A'BC'$ (rovnoram.)



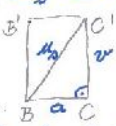
$x^2 = u_s^2 - \left(\frac{u}{2}\right)^2$ $u_s^2 = a^2 + v^2$

$x^2 = a^2 + v^2 - \frac{2a^2}{4}$

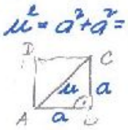
$x^2 = a^2 + v^2 - \frac{a^2}{2}$

$x^2 = \frac{a^2}{2} + v^2$

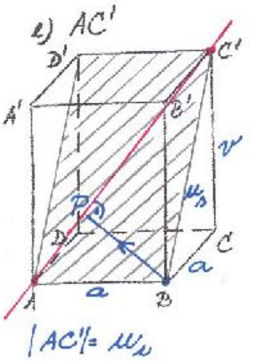
$x = \sqrt{\frac{16}{2} + 5{,}5^2} \doteq 6{,}2$ cm



$u^2 = a^2 + a^2 = 2a^2$

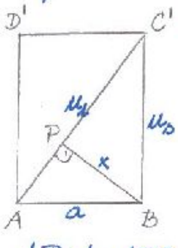


e) AC'



$|AC'| = u_t$

$|Bp| = |PB|$



! z rovnosti obsahů $\Delta ABC'$!

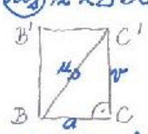
$\frac{1}{2} u_t \cdot x = \frac{1}{2} a \cdot u_s$

$x = \frac{a \cdot u_s}{u_t}$

$x = \frac{a\sqrt{a^2 + v^2}}{\sqrt{2a^2 + v^2}}$

$|Bp| = |PB| = x = \frac{4\sqrt{16 + 5{,}5^2}}{\sqrt{5{,}5^2 + 2 \cdot 16}} \doteq 3{,}45$ cm

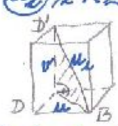
(u_s) z R$\Delta BCC'$



$u_s^2 = a^2 + v^2$

$u_s = \sqrt{a^2 + v^2}$

(u_t) z R$\Delta BDD'$



LÉPE na modelu

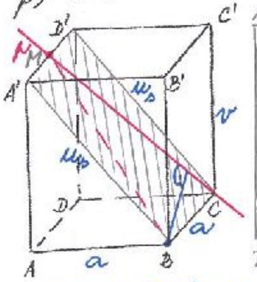
$[u^2 = a^2 + a^2 = 2a^2]$

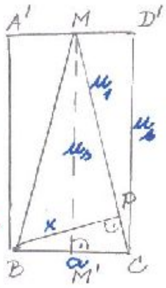
$u_t^2 = u^2 + v^2$

$u_t^2 = 2a^2 + v^2$

$u_t = \sqrt{2a^2 + v^2}$

f) CM





$|BP| = x$ z rovnoram. $\triangle BCM$

– ! z rovnosti obsahů !

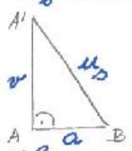
$\frac{a u_s}{2} = \frac{u_1 \cdot x}{2}$

$a u_s = u_1 x$

$x = \frac{a u_s}{u_1}$

$x = \frac{a\sqrt{a^2+v^2}}{\sqrt{\frac{5a^2}{4}+v^2}} = \frac{4\sqrt{4^2+5{,}5^2}}{\sqrt{\frac{5\cdot 4^2}{4}+5{,}5^2}} \doteq 3{,}84$ cm

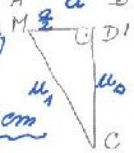
u_s: $u_s = |BA'| = |CD'| = |MM'|$



$u_s^2 = a^2 + v^2$

$u_s = \sqrt{a^2+v^2}$

u_1:



$u_1^2 = u_s^2 + \left(\frac{a}{2}\right)^2$

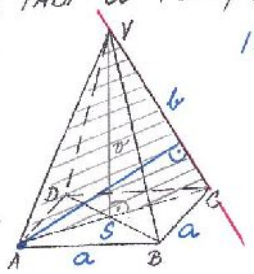
$u_1^2 = a^2 + v^2 + \frac{a^2}{4}$

$u_1 = \sqrt{\frac{5a^2}{4} + v^2}$

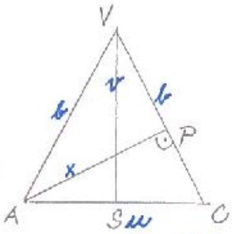
$|Bp| = |BP| = x$

② Urči vzdálenost vrcholu A pravidelného čtyřbokého jehlanu ABCDV od přímky CV, je-li

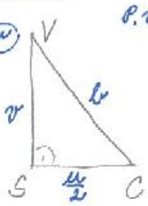
24/86 $|AB| = a = 4$ cm, $|AV| = b = 6$ cm.



$|AP|$ z rovnoram. $\triangle ACV$



v:



p.v. $b^2 = v^2 + \left(\frac{u}{2}\right)^2$

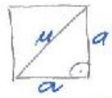
$v^2 = b^2 - \frac{u^2}{4}$

$v^2 = b^2 - \frac{2a^2}{4}$

$v^2 = b^2 - \frac{a^2}{2}$

$v = \sqrt{b^2 - \frac{a^2}{2}}$

u:



$u^2 = a^2 + a^2$

$u^2 = 2a^2$

$u = a\sqrt{2}$

$|Ap| = |AP| = x$

z rovnosti $S_{\triangle ACV}$

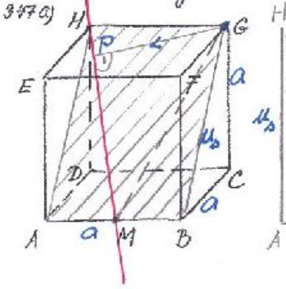
$\frac{1}{2} u \cdot v = \frac{1}{2} b \cdot x$

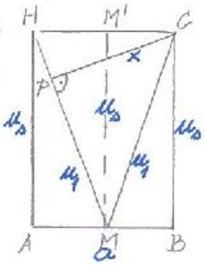
$x = \frac{u \cdot v}{b} = \frac{a\sqrt{2}\cdot\sqrt{b^2-\frac{a^2}{2}}}{b} = \frac{4\sqrt{2}\cdot\sqrt{6^2-8}}{6} \doteq 4{,}99$ cm $\doteq 5$ cm

DÚ (jako 1f)

③ Dána krychle ABCD s hranou $a = 4$ cm, M střed AB. Urči vzdálenost bodu G od ↔HM

3.47 a)





$\frac{1}{2} a u_s = \frac{1}{2} u_1 x$

$x = \frac{a u_s}{u_1}$

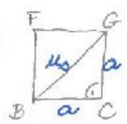
$x = \frac{a \cdot a\sqrt{2}}{\frac{3a}{2}}$

$x = \frac{2\sqrt{2}\cdot a^2}{3a}$

$x = \frac{2\sqrt{2}a}{3}$

$x = \frac{2\sqrt{2}\cdot 4}{3} \doteq 3{,}77$ cm

u_s:

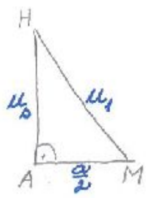


$u_s^2 = a^2 + a^2$

$u_s^2 = 2a^2$

$u_s = a\sqrt{2}$

u_1:



$u_1^2 = u_s^2 + \left(\frac{a}{2}\right)^2$

$u_1^2 = 2a^2 + \frac{a^2}{4}$

$u_1^2 = \frac{8a^2 + a^2}{4} = \frac{9a^2}{4}$

$u_1 = \frac{3a}{2}$

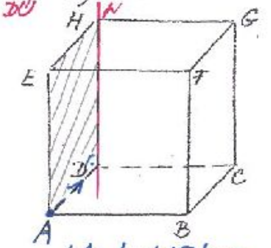
$|Gp| = |GP| = x$

z rovnosti $S_{\triangle MGH}$

④ Krychle ABCDEFGH s hranou délky $\underline{a}$. Urči vzdálenost bodu A od přímky

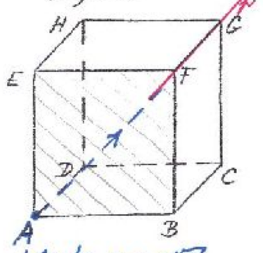
3.46 DÚ

a) DH



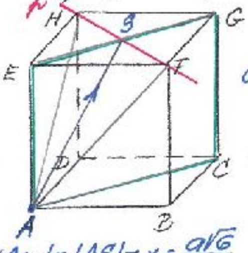
$|Ap| = |AD| = a$

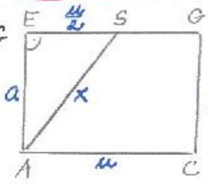
b) FG



$|Ap| = u = a\sqrt{2}$

c) FH jako 1d) (1.zp.) z obdél. ACGE





$x^2 = a^2 + \left(\frac{u}{2}\right)^2$

$x^2 = a^2 + \frac{u^2}{4}$

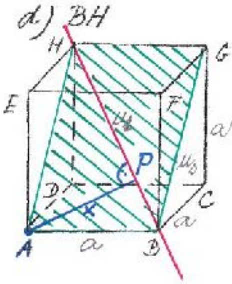
$x^2 = a^2 + \frac{2a^2}{4} = \frac{3a^2}{2}$

$x = \frac{a\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{a\sqrt{6}}{2}$

$|Ap| = |AS| = x = \frac{a\sqrt{6}}{2}$

(2.zp.) z rovnoram. $\triangle AFH$

d) BH



$|A \leftrightarrow BH| = |Ap| = |AP| = x$

z rovnosti $S_{\triangle ABH}$

$\frac{1}{2} x \cdot u_t = \frac{1}{2} a \cdot u_s$

$x = \frac{a \cdot u_s}{u_t}$

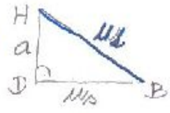
$x = \frac{a \cdot a\sqrt{2}}{a\sqrt{3}} = \frac{a\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{a\sqrt{6}}{3}$

(u_s)

$u_s^2 = a^2 + a^2 = 2a^2$

$u_s = a\sqrt{2}$

(u_t)



$u_t^2 = a^2 + u_s^2$

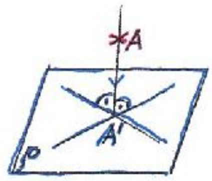
$u_t^2 = a^2 + 2a^2$

$u_t^2 = 3a^2$

$u_t = a\sqrt{3}$

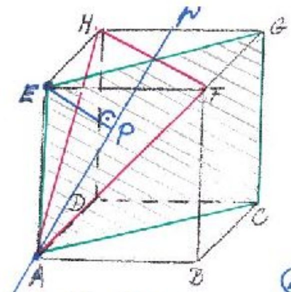
- VZDÁLENOST BODU OD ROVINY

- vzdálenost tohoto bodu a jeho pravoúhlého průmětu do této roviny



Příklady

⑤ Krychle, $|AB| = a = 5$ cm. Urči vzdálenost bodu E od roviny AFH



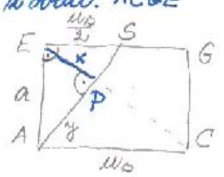
musíme určit pravoúhlý průmět bodu E do $\leftrightarrow AFH$

(tj. kolmici vedenou bodem E na rovinu AFH

1. bodem E sestr. kolmou rovinu k $\leftrightarrow AFH = \rho$ ... $\leftrightarrow ACE = \sigma$
2. určíme průsečnici p rovin ρ, σ
3. bodem E vedeme přímku $\perp$ na p ($k \perp p$)
4. $P \in k \cap p$

$|E\rho| = |EP| = x$

z obdél. ACGE



(EP) z rovnosti $S_{\triangle ASE}$

$\frac{1}{2}|EP| \cdot |AS| = \frac{1}{2}|AE| \cdot |ES|$

$x \cdot y = a \cdot \frac{u_s}{2}$

$x = \frac{a \cdot \frac{u_s}{2}}{y}$

$x = \frac{a \cdot \frac{a\sqrt{2}}{2}}{\frac{a\sqrt{6}}{2}} = \frac{a\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$x = \frac{a\sqrt{12}}{6} = \frac{a2\sqrt{3}}{6} = \frac{a\sqrt{3}}{3} = \frac{5\sqrt{3}}{3} \doteq 2{,}9\ \text{cm}$

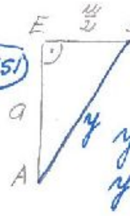
(u_s)

$u_s^2 = a^2 + a^2$

$u_s^2 = 2a^2$

$u_s = a\sqrt{2}$

($y = |AS|$)



Pyth. v.

$y^2 = a^2 + \left(\frac{u}{2}\right)^2$

$y^2 = a^2 + \frac{u^2}{4}$

$y^2 = a^2 + \frac{2a^2}{4}$

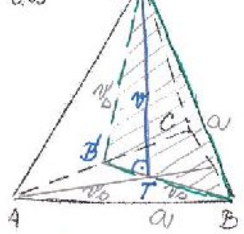
$y^2 = a^2 + \frac{a^2}{2}$

$y^2 = \frac{3a^2}{2}$

$y = \frac{a\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{a\sqrt{6}}{2}$

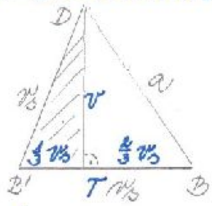
⑥ Pravidelný čtyřstěn ABCD, $a = 5$ cm. Urči vzdálenost bodu D od roviny ABC

3.55



stěny rovnostranné △

z rovnoramen. $\triangle BB'D$



$|D\rho| = |DT| = v$

$v_s^2 = v^2 + \left(\frac{v_s}{3}\right)^2$

$v^2 = v_s^2 - \frac{v_s^2}{9}$

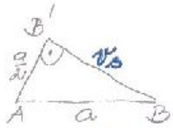
$v^2 = \frac{8v_s^2}{9}$

$v^2 = \frac{8 \cdot \frac{3a^2}{4}}{9} = \frac{6a^2}{9} = \frac{2a^2}{3}$

$v = \frac{a\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{a\sqrt{6}}{3}$

$v = \frac{5 \cdot \sqrt{6}}{3} \doteq 4{,}1\ \text{cm}$

(v_s) z rovnosti $\triangle ABC$



$a^2 = v_s^2 + \left(\frac{a}{2}\right)^2$

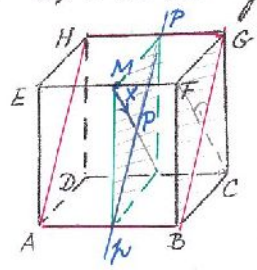
$v_s^2 = a^2 - \frac{a^2}{4}$

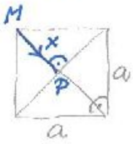
$v_s^2 = \frac{3a^2}{4}$

$\left[v_s = \frac{a\sqrt{3}}{2}\right]$

(7) Krychle, M střed EF, N střed FG, P střed GH, $a = 4$ cm. Urči vzdálenost bodu

3.56 a) M od roviny ABG





(1. zp.) $|M \leftrightarrow ABG| = |MP| = x$

$x = \frac{u}{2} = \frac{a\sqrt{2}}{2}$

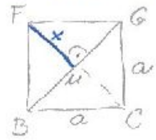
$x = \frac{4\sqrt{2}}{2}$ cm

$x = 2\sqrt{2}$ cm

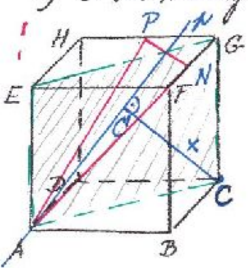
$x \doteq 2{,}8$ cm

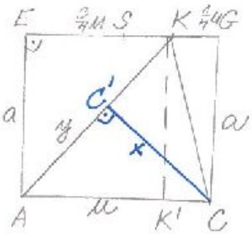
(2. zp.) s využ. ∥ rovin

$x = |M \leftrightarrow ABG| = |F \leftrightarrow ABG| = \frac{u}{2}$



b) C od roviny ANP





$x = |CC'| = |C \leftrightarrow ANP|$

$|CC'|$ z rovnosti $S_{\Delta ACK}$

$\frac{1}{2}|AC|\cdot|KK'| = \frac{1}{2}|AK|\cdot|CC'|$

$u \cdot a = y \cdot x$

$x = \frac{ua}{y}$

$x = \frac{a\sqrt{2}\cdot a}{\frac{a\sqrt{17}\cdot\sqrt{2}}{4}}$

$x = \frac{4a\sqrt{2}\cdot a}{a\sqrt{17}\cdot\sqrt{2}}$

$x = \frac{4a}{\sqrt{17}} = \frac{4\cdot 4}{\sqrt{17}}$

$x \doteq 3{,}9$ cm

($AK = y$) z PΔAEK

$y^2 = a^2 + (\frac{3}{4}u)^2$

$y^2 = a^2 + \frac{9u^2}{16}$

$y^2 = a^2 + \frac{9\cdot 2a^2}{16}$

$y^2 = a^2 + \frac{9a^2}{8}$

$y^2 = \frac{17a^2}{8}$

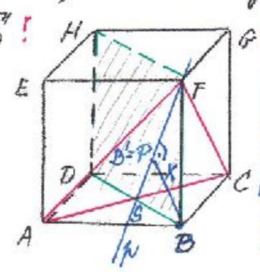
$y = \frac{a\sqrt{17}}{\sqrt{8}}$

$y = \frac{a\sqrt{17}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$y = \frac{a\sqrt{17}\cdot\sqrt{2}}{4}$

1. bodem C proložíme $\sigma \perp \rho$ ($\rho = \leftrightarrow ANP$, $\sigma = \leftrightarrow ACG$)
2. průsečnici $p = \leftrightarrow AK$
3. bodem C vedeme $k \perp p$
4. $C' \in k \cap p$
5. $|C\rho| = |CC'| = x$

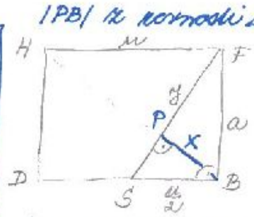
3.57 a)c) ! c) B od roviny ACF



1. bodem B $\sigma \perp \rho$; $\rho = \leftrightarrow ACF$, $\sigma = \leftrightarrow BDH$
2. průsečnici ρ, σ: $p = \leftrightarrow SF$
3. kolmici $k \perp p$ bodem B
4. $P \in k \cap p$
5. $|B\rho| = |BP|$

$|B \leftrightarrow ACF| = |BP|$

$|PB|$ z rovnosti ΔSPF



($y = |SF|$) z P.v.

$y^2 = a^2 + (\frac{u}{2})^2$

$y^2 = a^2 + \frac{u^2}{4}$

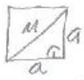
$y^2 = a^2 + \frac{2a^2}{4}$

$y^2 = \frac{3a^2}{2}$

$y = \frac{a\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{a\sqrt{6}}{2}$

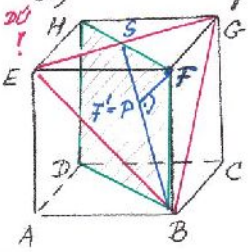
$\frac{1}{2}x\cdot y = \frac{1}{2}\cdot\frac{u}{2}\cdot a$

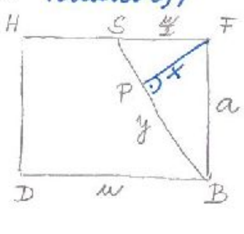
$x = \frac{\frac{u}{2}\cdot a}{y} = \frac{\frac{a\sqrt{2}}{2}\cdot a}{\frac{a\sqrt{6}}{2}} = \frac{a\sqrt{2}}{\sqrt{6}} = \frac{a\sqrt{2}}{\sqrt{2}\cdot\sqrt{3}} = \frac{a}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{a\sqrt{3}}{3}$

(u)  $u^2 = a^2 + a^2 = 2a^2$; $u = a\sqrt{2}$

$x = \frac{4\sqrt{3}}{3} \doteq 2{,}3$ cm

DÚ ! d) F od roviny BEG (obdoba c))





z rovnosti S_{ABSF}

$\frac{1}{2}a\cdot\frac{u}{2} = \frac{1}{2}\cdot x\cdot y$

$x = \frac{a\cdot\frac{u}{2}}{y} = \frac{au}{2y}$

$x = \frac{a\cdot a\sqrt{2}}{2\cdot\frac{a\sqrt{3}\cdot\sqrt{2}}{2}}$

$x = \frac{a}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{a\sqrt{3}}{3} = \frac{4\cdot\sqrt{3}}{3} \doteq 2{,}3$ cm

(u) $u^2 = a^2 + a^2$; $u^2 = 2a^2$; $u = a\sqrt{2}$

(y) $y^2 = a^2 + (\frac{u}{2})^2$

$y^2 = a^2 + \frac{u^2}{4}$

$y^2 = a^2 + \frac{2a^2}{4}$

$y^2 = a^2 + \frac{a^2}{2}$

$y^2 = \frac{3a^2}{2}$

$y = \frac{a\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$y = \frac{a\sqrt{3}\cdot\sqrt{2}}{2}$

($y = \frac{a\sqrt{6}}{2}$)

1. bodem F $\rho \perp \sigma$; $\rho = \leftrightarrow BEG$, $\sigma = \leftrightarrow BFH$
2. průsečnici ρ, σ: $p = \leftrightarrow BS$
3. bodem F kolmici k na p
4. $P \in k \cap p$
5. $|F\rho| = |FP|$